Impact of Limiting Hop Count on the Lifetime of Wireless Sensor Networks

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***Abstract*—In this study, we present a novel family of mixed integer programming (MIP) models to analyze the effects of limiting hop count on Wireless Sensor Network (WSN) lifetime. We performed analysis to uncover the trade-off between minimizing the number of hops and maximizing the network lifetime by exploring the parameter space through numerical evaluations of the optimization models. Our results reveal that minimum hop routing leads to significant decrease in network lifetime (up to 40%) when compared to the maximum network lifetime obtained without any restrictions on hop count. However, the decrease in network lifetime is negligible if the minimum hop routing criterion is modestly relaxed (*e.g.*, 3% decrease in network lifetime is possible if the minimum hop count is increased by 15%).**

***Index Terms*—wireless sensor networks, hop count, mixed integer programming, energy efficiency.**

I. Introduction

Wireless sensor networks (WSNs) consist of small form factor sensor nodes which are densely deployed over a geographical area to monitor physical phenomena such as temperature, pressure, humidity, etc. [1]. Prolonging network lifetime by efficient use of limited battery energy is a central issue in WSN design [2]. Communication related tasks dominate the energy budget of WSNs [3]. Communication energy consumption can be optimized through transmission power control, energy balancing routing, and sleep-wake up scheduling protocols.

Minimum hop routing is a widely used technique in WSNs [4]–[6], where sensor nodes send their data to the base station by using the path that is comprised of the minimum number of relay nodes (*i.e.*, the number of hops traversed by the packets are kept at the minimum). Furthermore, minimum hop routing approach is used as the baseline case for comparing the performance of WSN routing techniques in many studies [7], [8]. In fact, the common observation in these studies on minimum hop routing is that it is not possible to achieve the maximum network lifetime possible given a WSN deployment by using minimum hop routing. Indeed, there is a tradeoff between minimizing the hop count and maximizing the network lifetime in WSNs. By minimizing the hop count some nodes (positioned on minimum hop routes) are forced to carry excessive amount of traffic and run out of battery energy rapidly (*i.e.*, energy balancing to achieve the maximum network lifetime cannot be efficiently performed). By relaxing the minimum hop count constraint, overutilization of such nodes is mitigated. Nevertheless, the extent of the loss of lifetime due to minimum hop routing and the level of relaxation on hop count constraint to maximize network lifetime have never been investigated systematically in WSN literature.

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In this study, we built a mixed integer programming (MIP) framework to analyze the impact of limiting hop count on WSN lifetime. We seek the answers to the following research question through the numerical evaluations of the MIP models by exploring a large parameter space:

1) What is the extent of network lifetime decrease due to minimum hop routing with respect to the maximum network lifetime achievable without any constraints on network lifetime?
2) How much relaxation on hop count is necessary to achieve the maximum network lifetime?
3) Is it possible to achieve near maximum network lifetime by relaxing the hop count modestly?

II. Model

The network topology is represented as a directed graph $G = (V, A)$ where we define V as the set of all nodes including the base station (node-0). We also define set W which includes all nodes except the base station $W = V \setminus \{0\}$. $A = \{(i,j) : i \in W,\ j \in V - i\}$ is the set of arcs. Traffic generated at each node terminates at the base station either by direct transfer or through other sensors acting as relay nodes. The amount of traffic generated at node-k that flows from node-i to node-j is represented as an integer variable f_{ij}^k.

We adopt Mica2 motes' energy dissipation characteristics in our model [9]. Transmission power levels, transmission ranges, and energy dissipation amounts are presented in Table I. Energy dissipation for transmitting one bit of data at power level l and the maximum transmission range at power level l are denoted as $E_{tx}(l)$ and $R_{max}(l)$, respectively. If the distance between node-i and node-j (d_{ij}) is larger than $R_{max}(l)$ at maximum power level (*i.e.*, $d_{ij} > R_{max}(l_{max})$) then these nodes cannot communicate with each other which can be modeled as

$$f_{ij}^k = 0, \text{if } d_{ij} > R_{max}(l_{max}) \ \forall (i,j) \in A, \forall k \in W. \quad (1)$$

Note that, transmission power is not only discrete but also takes a value from a finite set denoted as S_L (*i.e.*, there are only 26 power levels to choose from). The optimal power level (l_{opt-ij}) to transmit over a distance d_{ij} is

$$l_{opt-ij} = \underset{l \in S_L,\ d_{ij} \leq R_{max}(l)}{\operatorname{argmin}} \left(E_{tx}(l) \right). \quad (2)$$

TABLE I: Transmission energy consumption ($E_{tx}(l)$ – nJ/bit) and transmission range ($R_{max}(l)$ – m) at each power level (l) for the Mica2 motes as a function of power level [9]. Energy dissipation for reception of data is constant ($E_{rx} = 922$ nJ/bit).

l	$E_{tx}(l)$	$R_{max}(l)$	l	$E_{tx}(l)$	$R_{max}(l)$
1 (l_{min})	671.88	19.3	14	843.75	41.19
2	687.50	20.46	15	867.19	43.67
3	703.13	21.69	16	1078.13	46.29
4	705.73	22.69	17	1132.81	49.07
5	710.94	24.38	18	1135.42	52.01
6	723.96	25.84	19	1179.69	55.13
7	726.56	27.39	20	1234.38	58.44
8	742.19	29.03	21	1312.50	61.95
9	757.81	30.78	22	1343.75	65.67
10	773.44	32.62	23	1445.31	69.61
11	789.06	34.58	24	1500.01	73.79
12	812.50	36.66	25	1664.06	78.22
13	828.13	38.86	26 (l_{max})	1984.38	82.92

For example, if the distance between node-4 and node-7 (d_{47}) is 20 meters then node-4 utilizes power level 2 (l_2) to transmit data on its link to node-7 (*i.e.*, $E_{tx}(l_{opt} - 47) = 687.50$ nJ/bit) because 19.30 m $< d_{47} \leq$ 20.46 m. We denote the transmission energy at power level l_{opt-ij} as $E^{opt}_{tx,ij}$.

The MIP model in Figure 1 is used to minimize the total hop count (β_{min}) of the network. Equation 3 states all data generated by sensor nodes terminate at the base station. In order to determine the total hop count, we assume that all sensor nodes create a single data packet. Hence, summation of flows would give the total hop count in the network. Equation 4 is used to guarantee that there are no loops. Flow conservation constraint at a particular source node-k_1 for flow $f^{k_1}_{ij}$ can be expressed as $\sum_{\substack{j\in V\\ i\neq j}} f^{k_1}_{k_1 j} = 1$. Note that the second term in Equation 3 is zero due to Equation 4 for this case. Hence, Equation 3 ensures that node-k_1 injects only one packet to the network as the source of flow $f^{k_1}_{ij}$. Flow conservation constraint at the base station for flow $f^{k_1}_{ij}$ can be stated as $\sum_{\substack{j\in W\\ i\neq j}} f^{k_1}_{j0} = 1$. By definition the base station does not send data to any other node, therefore, the first term in Equation 3 is equal to zero for this case. Hence, Equation 3, guarantees that the packet injected into the network by source node-k_1 for flow $f^{k_1}_{ij}$ terminates at the base station. Flow balance constraint at the nodes other than node-k_1 and the base station for flow $f^{k_1}_{ij}$ can be written as $\sum_{\substack{j\in V\\ i\neq j}} f^{k_1}_{ij} - \sum_{\substack{j\in W\\ i\neq j}} f^{k_1}_{ji} = 0$ which states that if the packet injected by source node-k_1 for flow $f^{k_1}_{ij}$ is received by another sensor node then it is forwarded out to the network (either to another relay node or to the base station). Equation 5 is used to calculate the total hop count in the network. Equation 6 states that all flows are non-negative.

The MIP model in Figure 2 is used to obtain the network lifetime (t) whilst a hop count limitation (β) is employed. Equation 7 is the flow balancing constraint. Each sensor node generates a single data packet at each round and during the network lifetime each node generates t amount of data packets (*i.e.*, network lifetime is given in terms of rounds). Equation 7 ensures that source node-k_1 for flow $f^{k_1}_{ij}$ injects t packets to the network, in total, and all these packets terminate at the base station. The difference of incoming and outgoing flows for all sensor nodes other than node-k_1 equal to zero (*i.e.*, no packets are lost on the path from the source node to the base station). Equation 9 states that the total energy dissipation at each node is limited by the amount of energy stored (e_i) in batteries (L_P represents the data packet size in bits). Equation 10 states that each sensor node is assigned equal initial energy (ξ) at the beginning of the network operation. Equation 11 limits the total hop count in network during lifetime. For example, if $\beta = 1.25\beta_{min}$ then the total hop count is allowed to take up to 25% more than the minimum hop count.

We can obtain the maximum network lifetime without any constraints on the hop count (t_{max}) by removing Equation 11 from the MIP problem in Figure 2. We will use t_{max} in our analysis to benchmark the impact of relaxing the minimum hop count constraint by increasing β. We are also interested in determining the hop count that gives t_{max} (β_{max}). To

Minimize β_{min}
Subject to:

$$\sum_{\substack{j\in V\\ i\neq j}} f^k_{ij} - \sum_{\substack{j\in W\\ i\neq j}} f^k_{ji} = \begin{cases} 1 & \text{if } i=k \\ -1 & \text{if } i=0 \\ 0 & \text{o.w.} \end{cases} \quad \forall i \in V, \forall k \in W \tag{3}$$

$$\sum_{j\in W} f^k_{jk} = 0 \quad \forall k \in W \tag{4}$$

$$\sum_{i\in W}\sum_{j\in V}\sum_{k\in W} f^k_{ij} = \beta_{min} \tag{5}$$

$$f^k_{ij} \geq 0 \quad \forall (i,j) \in A, \forall k \in W \tag{6}$$

Fig. 1: The MIP model to minimize total hop count

Maximize t
Subject to:

$$\sum_{\substack{j\in V\\ i\neq j}} f^k_{ij} - \sum_{\substack{j\in W\\ i\neq j}} f^k_{ji} = \begin{cases} t & \text{if } i=k \\ -t & \text{if } i=0 \\ 0 & \text{o.w.} \end{cases} \quad \forall i \in V, \forall k \in W \tag{7}$$

$$\sum_{j\in W} f^k_{jk} = 0 \quad \forall k \in W \tag{8}$$

$$L_P \sum_{k\in W} \left(\sum_{j\in V} E^{opt}_{tx,ij} f^k_{ij} + E_{rx} \sum_{j\in W} f^k_{ji} \right) \leq e_i \quad \forall i \in W \tag{9}$$

$$e_i = \xi \ \forall i \in W \tag{10}$$

$$\sum_{i\in W}\sum_{j\in V}\sum_{k\in W} f^k_{ij} \leq \beta \times t \tag{11}$$

$$f^k_{ij} \geq 0 \quad \forall (i,j) \in A, \forall k \in W \tag{12}$$

Fig. 2: The MIP framework to maximize lifetime of network with limited hop count

determine β_{max}, we replace the objective function in Figure 2 from maximization of t to minimization of β. Furthermore, t's are replaced with t_{max} values (*i.e.*, t_{max} is an input parameter but not an optimization variable anymore). Henceforth, β, which is an input parameter in the original MIP problem, becomes an integer variable and the solution of the modified problem gives us β_{max}.

In summary, to characterize the impact of hop count on network lifetime for a given network topology, we solve the optimization problem in Figure 1 to obtain the minimum hop count (β_{min}). The optimization problem in Figure 2 is solved to obtain the network lifetime when minimum hop count is enforced (*i.e.*, $\beta = \beta_{min}$). By solving the minimization version of the MIP problem in Figure 2, we obtain β_{max}. Note that the maximum network lifetime for a given topology without any adverse effects of hop count constraint can be obtained by solving the maximization problem in Figure 2 by setting $\beta \geq \beta_{max}$. By solving the original MIP problem in Figure 2 for a range of β values ($\beta \in [\beta_{min}, \beta_{max}]$), we determine the network lifetime as a function of hop count limit.

III. Analysis

We use MATLAB to construct the network topology, and General Algebraic Modeling System (GAMS) with CPLEX solver for the numerical analysis of the MIP models. We use a disk shaped network with radius R_{net} consisting of N sensor nodes which are randomly placed within the disk (*i.e.*, we used a uniform random distribution to determine the positions of sensor nodes). The position of each node is expressed in polar coordinates by its angle (θ_n) and its radius (r_n). The angle is determined from a uniform random distribution with limits $[0, 2\pi]$. To determine the radius, we first obtain a value, r_{n0}, from the random distribution with limits $[0, 1]$. We obtain the radius by using the following formula: $r_n = R_{net}\sqrt{r_{n0}}$. The base station is at the center. The results are averaged over 40 independent runs (*i.e.*, 40 randomly generated topologies). Data packet size and initial energy values are taken as $L_P = 1024$ bits and $\xi = 25$ KJ.

In Figure 3, we investigate the effects of N and R_{net} on β_{min} (Figure 3a), on lifetime reduction when the hop count is limited by β_{min} (Figure 3b), and on the increase in hop count necessary to achieve the maximum lifetime (Figure 3c). β_{min} increases as both N and R_{net} increases. Since β_{min} is the aggregated number of hops in the network, adding new nodes to the network brings more hops. Increasing R_{net} gives rise to β_{min} because the average number of hops to reach the base station gets larger.

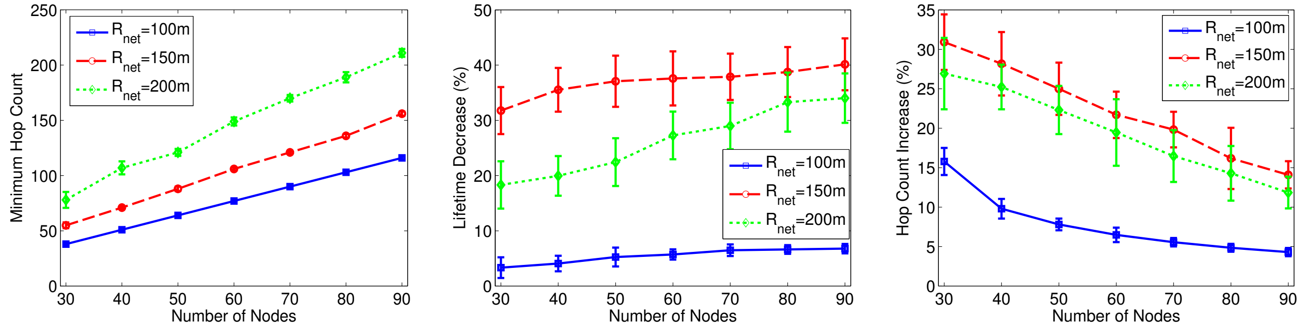
In Figure 3b, the decrease in network lifetime when compared to the unlimited hop count case is presented as a function of N (*i.e.*, the values on y-axis is the percentage ratio of t with β_{min} to t_{max} for each N value). The decrease in lifetime (with respect to t_{max}) increases as the number of nodes increases for all R_{net} values. Since the node density increases with increasing N, nodes have more ample routes to the sink, however, by limiting the usable paths by enforcing β_{min}, available energy balancing options cannot be used. As a general trend, percentage lifetime decrease for $R_{net} = 100$ m and $R_{net} = 200$ m scenarios are lower than the decrease with $R_{net} = 150$ m case. In sparser networks ($R_{net} = 200$ m), the decrease in lifetime is lower because even with the unlimited hop count case, the options of energy balancing are limited in sparser networks, whereas, in denser networks ($R_{net} = 100$ m) energy imbalance caused by the minimum hop count constraint is not as severe as the sparser networks because nodes can find more energy efficient paths without increasing the minimum hop count.

In Figure 3c, the percentage difference between β_{max} and β_{min} is presented as a function of N. Both the larger ($R_{net} = 200$ m) and smaller ($R_{net} = 100$ m) networks has lower percentage increase in hop count when compared to the percentage hop count increase for $R_{net} = 150$ m case due to the reasons explained in the preceding paragraph (node density decreases for increasing R_{net} when the number of nodes is fixed). However, the differences between different R_{net} scenarios in Figure 3c are lower than the differences in Figure 3b because a relatively smaller percentage increase in hop count leads to larger percentage increase in lifetime, especially in larger networks as illustrated in Figure 4b and Figure 4c. For example, in Figure 4c, lifetime improvements obtained by increasing the hop count from 0% to 10% for networks with $N = 30$, $N = 60$, and $N = 90$ are 14.27%, 20.46%, and 33.98%, respectively.

In Figure 4, the decrease in network lifetime (with respect to t_{max}) is presented as a function of percentage increase in the hop count (0% corresponds to β_{min}). The required increase in hop count never exceeds 15% to achieve the 95% of the maximum achievable lifetime regardless of R_{net} and N. Indeed, in dense networks it is possible to reach t_{max} exactly with only 5% increase in β_{min} (*e.g.*, $R_{net} = 100$ m and $N = 90$).

One interesting point to mention is that as the hop count increases from 0% to 5%, the order of lifetime decrease changes for all R_{net} values (e.g., percentage decrease for $N = 30$ is not always lower than percentage decrease for $N = 60$). However, the increase in lifetime due to hop count increases consistently higher for denser networks. For example, in Figure 4c, lifetime improvements obtained by increasing the hop count from 0% to 5% for networks with $N = 30$, $N = 60$, and $N = 90$ are 10.58%, 11.64%, and 27.30%, respectively. Likewise, improvement in lifetime values by increasing the hop count from 0% to 10% for networks with $N = 30$, $N = 60$, and $N = 90$ are 14.27%, 20.46%, and 34.16%, respectively. The reason for such behavior is that denser networks can use relatively lower percentage increases in hop count more efficiently than sparser networks due to more ample availability of alternative routes for energy balancing when compared to sparser networks.

The results of our analysis are very useful in practical WSN design and implementation. We have shown that there is a tradeoff between the hop count and network lifetime. In fact, it is not possible to achieve the maximum network lifetime without relaxing the minimum hop count constraint. However, the extent of the relaxation is limited. Indeed, by increasing the hop count, on the average, less than one hop per each route (from each source node to the base station), it is possible to achieve the maximum network lifetime (the maximum value of

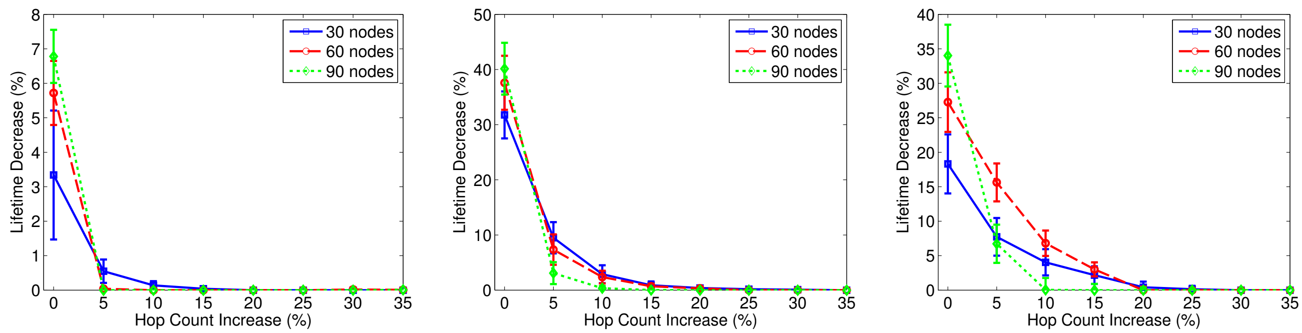


(a) Minimum hop count.

(b) Decrease in network lifetime when the minimum hop count is enforced.

(c) Hop count increase to achieve the maximum lifetime.

Fig. 3: Trade-off between network lifetime and hop count as a function of N (error bars show the 95% confidence intervals).



(a) $R_{net} = 100$ m.

(b) $R_{net} = 150$ m.

(c) $R_{net} = 200$ m.

Fig. 4: Change in network lifetime due to the relaxation of the hop count constraint (error bars show the 95% confidence intervals).

the average hop count increase per source to base station route is 0.7, which is obtained with $N = 30$ and $R_{net} = 200$ m). Therefore, after determining the minimum hop routes from each sensor node to the base station, a local search algorithm with only a few hop search radius (mostly limited by one hop) can determine the optimal paths for network lifetime maximization.

IV. Conclusion

In this study, we present a novel MIP framework to analyze the impact of limiting hop count on WSN lifetime under optimal operating conditions. We explored the parameter space through numerical evaluations of the MIP models to characterize the effects of minimum hop routing on network lifetime. Furthermore, we investigate the extent of relaxation on minimum hop count constraint to achieve the maximum network lifetime. Our results show that limiting the hop count to the minimum value has significant effects on network lifetime (*e.g.*, minimum hop routing can result in more than 40% decrease in network lifetime with respect to the network lifetime obtained without limiting the hop count). To achieve the maximum network lifetime, up to 32% increase of the minimum hop count can be needed depending on the network area and number of nodes in the network. However, it is possible to obtain insignificant lifetime decreases with moderate increases in the minimum hop count (*e.g.*, at most 7% decrease in network lifetime with 10% increase in the hop count).

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